# Functions

15 June 2008

# Unit 2 Functions

# Introduction

We draw graphs to represent functions o formulae and this depends on the formation of Cartesian axes. These were first introduced by the French philosopher René Descartes (1596-1650), from which the name "Cartesian" is derived.

The link between algebra and geometry seems quite natural now but the subject appeared in the 17 th Century with Pierre Fermat and René Descartes.

Before this time, geometry had been studied for many centuries, so that the properties of particular curves had been well understood.

Algebraic notation and analysis particularly related to the solving of equation had also been progressing. What Fermat and Descartes did was to take an algebraic equation and plot corresponding points on a rectangular grid combining geometry and algebra together.

Nowadays, it seems entirely natural to draw graphs of functions given in algebraic terms.

### Contents

- Finding coordinates that satisfy a rule.
- Plotting graphs of simple equations using a table of values.
- Knowing some properties
- Reading information off a graph
- Constructing functions arising from real-life problems given by values tables, sentences or algebraic expressions
- Using graphs to solve simple problems involving direct and inverse proportion

The water at the top of the falls has potential energy due to its height . As the water flows over the edge of the falls its energy changes to kinetic energy. The water flows through a turbine which converts the kinetic energy into mechanical work used to drive an electric generator.

There is a **relationship** between the **heigh**t and the **potential energy** 







Finding	perform a functio	ed on the input,ar on table like this o	nd the output(the v ne	alue of the functior	ns 1)into	
Values for		Input or domain	Rule	Output or range		
Functions		x	3x-1	f(x)		
	The domain contains all the values of x and the range contains all					
	the value	es of f(x)				

Input x	Rule 3x-2	Output f(x)
-1	3(-1)-2	-5
0	3(0)-2	-2
1	3(1)-2	1
2	3(2)-2	4

**Example**. Complete the function table. Replace x in the rule with each input value.

#### ACTIVITY 1.

a) Complete this function table.

x	-5x	f(x)

Find each function value.

• $f(x) = x-3$ $f(6) = \dots$ f() = -5	$f(-1) = \dots f(-1) = 0$	$f(1.4) = \dots f(1.4) = 1.5$	f(0) = f ( ) =8
■ g(x)=x+5 g(-3) = g( ) =-7	g(1) = g( ) =0	g(0.4) = g( ) =2.5	g(0) = g( ) =6
<ul> <li>h(x)=-x-3</li> <li>h(2) =</li> <li>h( ) =-4</li> </ul>	h(-2) = h( ) =0	h(1.5) = h( ) =3.5	h(0) = h( ) =7

# The Coordinate Plane

You can graph points in a plane on a coordinate system



#### ACTIVITY 2. On-line lesson (with sound) CARTESIAN PLANE

A **linear function** is a relationship between two variables x,y defined by the rule y=mx+n Its graph is a **straight line** 

Graphing a Linear Function	To graph a linear function,make a function table. List at least three values for x. Graph each ordered pair. Connect the points with a straight line.
Remember	The x-axis is the horizontal axis and the y-axis is the vertical axis. We graph the independent variable on the x-axis and the dependent variable on the y-axis.

## **Example**. Graph the function y = 3x - 2.

First, we choose some values for x, and find the matching values for y and we make a table to show the ordered pairs.



Then, we graph the ordered pairs from the table and we draw the line that joins these points. This line is the graph of y = 3x - 2.

ACTIVITY 3. Graph each function.

- a) y=x-10
- b) y=-x
- c) y=5-2x

ACTIVITY 4.Interactive exercise to fill in correct y for each x FINDING VALUES

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**Y-intercept** is the point at which the function intersects the Y-axis and **X-intercept** is the point at which the function intersects the X-axis.

A function is **increasing** if the y variable increases as x grows. It's **decreasing** if the y variable decrease as x grows.

The point on the graph where the curve changes from an increasing curve to a decreasing curve is a **relative maximum**. If this point is the highest on the graph we say that it's a **absolute maximum**. The point on the graph where the curve changes from an decreasing curve to a increasing curve is a **relative minimum**. If this point is is the lowest on the graph, we say that it's a **absolute minimum**.



#### ACTIVITY 5.

Find the X-intercept and the Y-intercept on the graphs in the **ACTIVITY 3** and explain if they are increasing or decreasing

ACTIVITY 6.-Interactive exercise (with sound) PLOTTING A STRAIGHT LINE

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The slope or gradient of a straight line y=mx+n is the change in y with respect to the change in x ."m" represents the gradient and "n" is the y-intercept. Slope is a number that tells how steep the line is. Vertical Change in y Slope= change :rise Change in x Horizontal change :run The slope is the same for any two points on a straight line. A line with a positive slope rises to the right. A line with a negative slope falls to the right. Example. The graph y = 3x + 1 has a gradient of 3 and a yintercept of 1. m=3, n=1 We can use this information to plot the graph, without drawing a table y = 3x + 1 Change in y function Change in x domain,range input,output table function  $m = \frac{3}{1} = 3$ ,slope,gradient equation, straight line n=y-intercept=1 **ACTIVITY 7. Power point Presentation** GRADIENT ACTIVITY 8. Complete the table: Equation Gradient Y-intercept y = -3x + 2y = 2x - 45 4 3 -1

# ACTIVITY 9.

Determine the equation of each of the following straight lines







ACTIVITY 10.Game to interpret a football distance/time graph DISTANCE TIME GRAPH

# ACTIVITY 11.

At a car dealership, a salesman worked for three days. On the first day he sold 5 cars. On the second day he sold 3 cars. On the third day he sold 8 cars.

a) Make a table showing the number of cars sold for each day.



b) Complete and write the data as a set of ordered pairs.

The day is the variable and the number of cars is the

variable.

The ordered pairs are (1, ), (2, ), and (3, )

c) Draw a graph that shows the relationship between the day and the number of cars sold

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# ACTIVITY 12.

A teacher is taking his classes to the cinema. The price is 5 Euro per student, and at most,120 students will go.

a) Identify a reasonable domain and range for this situation.

The domain contains the number of students that are going to the cinema Therefore, a reasonable domain would be values from ...... to......students The range contains the total price

Thus, a reasonable range is from......to......

- b) Make a values table
- c) Graph the data
- d) Is the relation a function?

### ACTIVITY 13.

The table shows the percent of people satisfied with the way things were going in a country at the time of the survey.

Year	1992	1995	1998	2001
Percent Satisfied	21	32	60	51

a) Determine the domain and range of the relation.

The domain is

The range is

b) Graph the data.

The values of the x-axis need to go from 1992 to 2001. Begin at 1992 and extend to 2001 to include all of the data.

The units can be 1 unit per grid square.

The values on the y-axis need to go from 21 to 60.Begin at 0 and extend to 70. You can use units of 10.

c) What conclusions might you make from the graph of the data? People became more satisfied with the country from to to to be the percentage dropped from to to to be the percentage dropped from the percentage

#### ACTIVITY 14. Revise your vocabulary

Choose a word in the box and fill the blanks below:



#### ACTIVITY 15. Revise your vocabulary

Write the sentences in the correct order:

- the function /X-intercept /is/at which/intersects/the point/X-axis
- the point / is /at which/ Y-intercept/ the function/ the Y-axis/ intersects
- increasing / the y variable /is/ increases /A function /as x grows /if
- as x grows/ the y variable / It's /decrease / decreasing / if.
- on the graph/absolute maximum/a/The highest point/is/.
- /absolute minimum/a/The lowest point/is/ on the graph.

#### ACTIVITY 16. Population

The table shows the total annual growth rate (%) from 1955 to 2005 in the least developed countries

Year	Total annual growth rate
1955	2.00
1960	2.21
1965	2.37
1970	2.53
1975	2.52
1980	2.49
1985	2.55
1990	2.62
1995	2.69
2000	2.45
2005	2.42

a) Complete the text

The domain is.....The range is....

b) Graph the data

The values of the x-axis need to go from 1955 to 2005. The units can be 1 unit per grid square.

The values on the y-axis need to go from 2 to 2.42. Begin at 2 and use units of 0.1

- c) What was the total annual growth rate in 1982?
- d) In which year was the total annual growth 2.5?
- e) Write the intervals on which the function is increasing and the intervals on which the function decreasing:

Increasing	(	,	)	(	,	)
Decreasing	(	,	)	(	,	)

f) Write the relative maximum and minimum

Relative maximum	(	(,)	(	( ,	)
Relative minimum	(	, )	(	( ,	)

The function has a absolute ..... at ( , ). The function has a absolute maximum at ( , )

g) When was the growths faster: from 1955 to 1975 or from 1980 to 1995?

#### ACTIVITY 17. Population

The table shows the total annual growth rate (%) from 1955 to 2005 in more developed regions

Year	Total annual growth rate
1955	1.20
1960	1.17
1965	1.08
1970	0.84
1975	0.77
1980	0.66
1985	0.58
1990	0.60
1995	0.45
2000	0.32
2005	0.36

a) Graph the data on the same diagram in the previous activity

b) Complete the text

- 1960 to 1965 than between 1990
- c) Use the graph where the two tables are represented to answer these questions:
- Which of the regions or countries (least developed or more developed) had the highest growth rate in 1980?
- In which years the growth rate was the same in more developed regions as in the least developed countries ?



# ACTIVITY 18.

In forests in many warm and humid parts of the world, we can hear crickets making chirping sounds. The rate which they chirp depends

on the temperature. A formula that connects the temperature in grades Fahrenheit,y,and the number of chirps made in a minute,x, is

y=0.25x+40

- a) What is the temperature when the number of chirps is :60, 80, 100?
- b) Plot the graph of the function using axes 0 < x < 120, 0 < y < 100
- c) Use your graph to predict:



- The temperature when there are 71 chirps per minute
- The number of chirps per minute when the temperature is 67° F
- d) What is the relationship between the temperature in degrees Celsius and the number of chirps per minute?

#### ACTIVITY 19.

The charges of a move firm consist of a fixed charge of 75 Euro, and a variable charge of 10 Euro per kilometre travelled.

- a) Identify the independent variable and the dependent variable.
- b) Make a value table
- c) Draw a graph of this relationship for 0 < x < 50
- d) Use the graph to estimate the distance travelled when the cost is 180 Euro
- e) Write the formula for the total cost, y, in terms of the distance travelled, x km

# ACTIVITY 20.

The formula for the volume of a rectangular prism whose base has an area of 8  $cm^2$  is V=8h,where V is the volume and h is the height.

- a) Graph the function
- b) What is the domain?.What is the range?
- c) What is the volume of a rectangular prism whose base has an area of 8 square and whose height is of 5 cm.
- d) What is the height when the volume is 35cm<sup>2</sup> ?

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### ACTIVITY 21.

The graphic below shows the variations in Earth's surface temperature from the year 1860 to 2000



a) Make a function table

х	у
1880	
1920	
1945	
1990	
	-0.1
	0.4

b) Complete the text:

The independent variable is.....and the dependent one

is.....

The domain is......and the .....is [-0.1,0.7]approximately. The function has a absolute ...... at (1865,...). The function has a absolute maximum at ( ... ,0.7)

c) Write three intervals on which the function is increasing and three ones on which the function is decreasing:

Increasing	( ,	)	(		)	(	( ,	)
Decreasing	( ,	)	(	,	)	(	( ,	)

d) How many degrees did the the world heat up last century, approximately?

e) When was the growth faster: from 1920 to 1940 or from 1980 to 2000?

#### ACTIVITY 22 . Potential energy

Two water slides in a Aquatic Park are 50 m high. A 45-kg girl is in resting state at the top of a slide and begins to goes down it.

The table shows how the gravitational potential energy varies according to the height (We suppose the water slide is frictionless)

Height h	Potential energy PE
50	22050
25	11025
10	4410
5	2205
2	882
1	441
0,5	220,5
0,25	110,25
0,0000050	0,002205



a) Complete the text:

The independent variable ,h,is......It's measured in...... and the dependent one,PE, is......in joules.

The domain is ......The range is.....

b) Draw the graph.

The values on the x-axis goes from ...... to ....... The units can be ...... unit per grid square.

The values on the y-axis goes from ...... to ...... Begin at ..... and use units of .....per grid square.

- c) Use the graph to find approximately
  - the potential energy when the height is 30 m
  - the height when the potential energy is 1000 Joules
- d) The equation that describes this relationship is

In general,

#### Gravitational potential energy=PE=m.g.h

where

• EP = Energy (in Joules)

- · m = mass (in kilograms)
- $\cdot$  g = gravitational acceleration of the earth (9.8 m/s<sup>2</sup>)
- · h height above earth's surface (in metres)

- e) Use the equation to find the exact value of the amounts in the exercise c)
  - PE =..... when h=30 m
  - h=..... when PE=1000 Joules
- f) What is the kinetic energy when the girl reaches the bottom of the slide?

ACTIVITY 23. Potential energy

A 55-kg boy is also in resting state at the top of the another slide and begins to goes down it at the same time as the girl.

a) Make a values table(use a spreadsheet)

Height	Potential energy
50	
25	
10	
5	
2	
1	
0,5	
0,25	
0,0000050	

- b) Graph the data on the plane coordinate in the ACTIVITY  $\mathbf{22}$
- c) Use the tables to find:
  - The change in gravitational potential energy when the boy slides from the point 25 m high to the point 5 m
  - The change in gravitational potential energy when the girl slides from the point 25 m high to the point 5 m
- d) Use the graphs to find approximately:
  - The change in gravitational potential energy when the boy slides from the point 30 m high to the point 6 m
  - The change in gravitational potential energy when the girl slides from the point 30 m high to the point 6 m
- e) What is the kinetic energy when the girl reaches the point 10 m high?
- f) What is the answer in the case of the boy?

### ACTIVITY 24. Kinetic energy

This graph shows the relationship between the kinetic energy and the speed of a car with a mass of 1400 kg and the relationship between the kinetic energy and the speed of a bike with a mass of 200 kg



(Each grid on the y-axis has 10000 units of Joules)

a) Make a values table of each function Example:

(	Car	В	ike
Speed(m/s)	Kinetic energy(Joules)	Speed(m/s)	Kinetic energy(Joules)
10	70000	10	
20	280000	20	
30	630000	30	
40		40	
50		50	

b) Complete the text: The independent variable ,v,is......lt's measured in...... and the dependent one,KE, is.....in Joules.

The domain is ......The range is.....

- c) Use each graph to find approximately
  - the kinetic energy when the speed is 90 km/h
  - Before looking at the graph, you must solve: 90 km/h=.....m/s
  - the speed when the kinetic energy is 1900000 Joules

d) The equation that describes the relationship between the speed and the kinetic energy of the car is :

$$\frac{KE = \frac{1}{2}1400.v^2}{KE = 700.v^2}$$

In general,

#### Kinetic energy=KE= $\frac{1}{2}$ . m. v<sup>2</sup>

where

- KE = Energy (in Joules)
- · m = mass (in kilograms)
- $\cdot v = speed$  (in m/sec)

e) What is the equation that describes the relationship between the speed and the kinetic energy of the bike ?

- f) Use the equation to find the exact value of the amounts in the exercise c)
  - KE = ..... when v=90 km/h
  - v=..... when KE=1900000 Joules

g) Use a spreadsheet or a graphing program to check the values of the tables in the exercise a)

h) What happens to the amount of kinetic energy when the speed doubles?

#### ACTIVITY 25. Kinetic energy

Write an equation to describe the relationship between the speed of a 400-kg motorbike and the kinetic energy

- a) Make a values table (use a spreadsheet or a graphing program)
- b) Graph the data on the plane coordinate in the previous ACTIVITY
- c) Use the graph to find
  - the kinetic energy when the motorbike moves at120 Km/h=.....m/sec
  - the speed when the kinetic energy is 600000 Joules
- d) Use the equation to find the kinetic energy when the motorbike moves at 120 Km/h=.....m/s

Two variables x e y are in direct proportion when one increase as the other increase by the same percentage. We describe a direct variation by an equation of the form y=k.x, where  $k \neq 0$ .We say y is directly proportional to x. k is the constant of variation or proporcionality Two variables x e y are in inverse proportion when one increase as the other decrease . We describe a inverse variation by an equation of the form  $y = \frac{1}{k}$ .x, where  $k \neq 0$ .We say y is inversely proportional to x .k is the constant of variation or proporcionality

#### Examples

- If you buy 1kg of strawberries you pay 1,75 Euro and if you buy 2 kg you pay 2.1.75=3,5 Euro. The amount that you pay,y, is in the same proportion than the amount that you buy, x. These variables are in direct variation (y is directly proportional to x) and y=1,75.x is the equation that describes it.
- If you consume 21 m<sup>3</sup> of water you pay 5,91 Euro and if you consume 42 m<sup>3</sup> you pay 16,03 Euro. The consume of water, x, and the cost,y, aren't in the same proportion. These variables aren't in direct variation (y isn't directly proportional to x)
- If a car moves at 60 km/h, it takes 1 hour to travel a distance of 60 kilometres. If the car moves at 120 km/h, it takes 0,5 hour to travel the same distance. For that constant distance, the speed ,x,and the time,t, **are in inverse variation(y is**

directly proportional to x) and  $y = \frac{60}{x}$  is the equation that describes it

#### ACTIVITY 26.

The diameter of a tree trunk varies directly with the age of the tree. A 45-year-old tree has a trunk diameter of 45,72 cm.

- a) Make a value table of the function that relates the age of the tree and the diameter of its trunk
- b) Graph the function
- c) Write the algebraic expression of the function
- d) What is the age of a tree that has a trunk diameter of 50,8 cm

#### ACTIVITY 27.

When a diver goes underwater, the weight of the water exerts pressure on the diver. The table below shows how the water pressure on the diver increases as the diver's depth increases.

Diver's Depth (in feet)	Water Pressure (in pounds per square inch)
10	4.4
20	8.8
30	13.2
40	17.6
50	22.0

- a) What is the water pressure on a diver at a depth of 60 feet?Explain how you obtained the answer.
- b) What is the water pressure on a diver at a depth of 100 feet?Explain how you obtained the answer.
- c) Write an equation that describes the relationship between the depth, D, and the pressure, P.
- d) Use your equation from part c) to determine the depth of the diver, assuming the water pressure on the diver is 46.2 pounds per square inch.
- e) Did you know that "pound" is a measure of weight that is used by many people in Britain instead of "kilogram" and "inch" and "feet" are measures of length that are equivalent to 2,54 centimetres and 30,48 centimetres respectively? Use the equivalences 1 pound=0.454 kg ,1 inch=2,54 cm ,1 feet =30,48 cm to make the table from part a) taking kilogram(force) per square centimetre as unit for Water Pressure instead of pounds per square inch and metre as unit for Depth instead of feet

Example:



### ACTIVITY 28.

A factory produces 1500 can of juice in 3 hours.

- a) Make a value table of the function that relates the amount of can produced and the time
- b) Draw the graph
- c) Write the equation that describes this relationship

#### ACTIVITY 29.

A manufacturer of postcards decides to experiment with cards of different shapes.

The cards should be rectangles with an area of 120  $\rm cm^2. The height of the cards is y cm and the width is x cm$ 



a) Complete the table:

x	5	10	15	20	25	30
у		12				4

b) Is the height inversely proportional to the width? Explain the reason c) Draw the graph d) Use your graph to find the width of a postcard that has a height of 7 cm e) If the height of a postcard must be no greater than 14 cm, what is the least width it increasing can have? decreasing maximum minimum continuous direct inverse proportion





#### ACTIVITY 30.

Determine the points where the following function isn't continuous and explain the reason.



### ACTIVITY 31

Imagine that the following graph shows the relationship between the consume of water (m<sup>3</sup>) and the cost(Euro)

+1	50												
+1	90-												
+5													
+1													
	г	+3	30	+	50		+1	00		+1	.50		×

- a) Identify the dependent variable and the independent variable
- b) Make a value table
- c) Use the graph to complete the text:
   If you consume 30 m<sup>3</sup> of water,you will pay.....Euro. If you consume 60 m<sup>3</sup> of water,you will pay.....Euro. Therefore ,the consume of water and the cost...... in direct proportion.
- e) Write another points of discontinuity of the function.
- f) How many cubic metres of water did you consume last period if you paid 80 Euro?

# ACTIVITY 32

This graph shows the real relationship between the consume of water( m<sup>3</sup> )and the cost(Euro)



- a) Identify the dependent variable and the independent variable
- b) Make a value table

In your opinion, why is the price of water established in this way?

- e) Is there any point of discontinuity in this function?
- f) How many cubic metres of water did you consume last period if you paid 80 Euro?

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# Assessment

# **Revision and Revision Test**

Plane Coordinates Finding values	<ul> <li>On-line lesson (with sound) <u>PLOTTING POINTS</u></li> <li>Test <u>FINDING VALUES</u></li> </ul>
Drawing graphs Reading information	<ul> <li>Test <u>SITUATIONS</u></li> <li>Test <u>DISTANCE-TIME</u></li> </ul>
Linear functions (slope,y-intercept)	<ul> <li>Power Point Presentation         <u>STRAIGHT LINES</u>         Test         Worksheet (ACTIVITY 33)</li> </ul>
Functions (graph,table,rule,sentence) Properties	<ul> <li>Test         <u>FUNCTIONS</u>         Worksheet(ACTIVITY 34,35)</li> </ul>

# ACTIVITY 33.-Card Sort Activity

It can be used as a Worksheet or as Game

If is used as a game, students will work in groups of three or four to match the graph of he function with its corresponding equation. Each group must justify their responses.

y=x+3	x+y=3	y=2x-3
y=0,5x+3	y=3	3x+4y=12
y=3x	y=2x+3	y=x <sup>2</sup> +3

#### Answer Sheet

Graph of function	Equation	What strategy did you use to match them? Which line isn't the graph of a function? Which function isn't a linear function? What type of function is?	Key words and expressions
A			X-intercept
В			Y-intercept
С			slope
D			
E			
F			
G			
н			
I			

# Graph A



# Graph B



# Graph C





# Graph E









# Graph H







#### ACTIVITY 34.

The graph shows the distance travelled by a car on a journey to work Each grid on the y-axis represents 50 metres and each grid on the x-axis represents 10 seconds

a) Find the distance travelled by the car after:



Independent variable/Unit	
Dependent variable/Unit	
Domain	
Range	
Maximum/Minimum	
Increasing/Decreasing	
Continuous/Discontinous	

- c) The car stopped at two sets of traffics lights. How long did the car spend waiting at each set of lights?
- d) On which part of the journey did the car travel faster?
- e) How far did the car travel?
- f) How long did it take for the whole journey?
- g) What was the car's average speed for the whole journey?

### ACTIVITY 35.

Mary goes to the shop to buy a loaf of bread

The shop is 800 m from the house

She leaves home at 17:12 and walks to the shop at a ready speed

She takes 16 minutes to reach the shop and then 5 minutes to buy a loaf of bread Then she walks home at a ready speed arriving at 17:48

a) Draw a distance- time graph to represent her journey



# **Progress Sheet**

#### UNIT 2.- FUNCTIONS

By the end of this unit we must cover the following areas.

Objective	Level	Can	More	Date
		Do?	Practice?	
This is what you should already know :				
Generate coordinate pairs that satisfy a simple linear rule				
Construct and solve simple linear equations with integer coefficients using an appropriate				
method				
Break a calculation into simpler steps, choosing and using appropriate and efficient				
operation methods and resources, including computers.				
Read values from a graph.				
Plot a graph				
Find values of functions.				
Identify and construct functions given by a rule				
, a sentence ,a graph or an algebraic				
expression.				
Discuss and describe properties of a function.				
Draw and interpret graphs from real life.				
<ul> <li>a sentence ,a graph or an algebraic expression.</li> <li>Discuss and describe properties of a function.</li> <li>Draw and interpret graphs from real life.</li> </ul>				