SIMPLE AND QUADRATIC EQUATIONS

3rd COURSE .BILINGUAL-SECTION

SCHOOL YEAR 2009/2010

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1. INTRODUCTION:

Algebra is a branch of mathematics concerning the study of structure, relation, and quantity. Together with geometry, analysis, combinatory, and number theory, algebra is one of the main branches of mathematics. In secondary education we only study Elementary Algebra, so we need an introduction to the basic ideas of algebra, although this branch of mathematics is much broader. In addition to working directly with numbers, algebra covers working with symbols, variables, and set elements.

While the word *"algebra"* comes from Arabic word Al-Jabr (it means "reunion"), its origins can be traced to the ancient Babylonians, who developed an advanced arithmetical system with which they were able to apply formulas and calculate solutions for unknown values for a class of problems typically solved today by using simple and quadratic equations. By contrast, most Egyptians, Indian, Greek and Chinese mathematicians of this era (the first millennium BC), usually solved such equations by geometric methods. Later, Arab and Muslim mathematicians developed algebraic methods to a much higher degree of sophistication. In fact, the Arab mathematician Al-Khwarizmi was the first to solve quadratic equations using general methods, besides of a lot of other more difficult equations. This is the reason by which Al-Khwarizmi is considered the "father of algebra".

From then until nowadays, a lot of mathematicians have studied equations and algebraic problems in everywhere in the world, getting that Algebra is in a steady evolution. Very important results have been found and applied in all the branches of the mathematics and other sciences.

2. OBJETIVES:

In this unit you will learn to:

- Differentiate between identities and equations.
- Identify the equation's elements.
- Solve simple equations and quadratic equations (complete and incomplete) with one unknown quantity.
- Solve real-life problems using algebraic language and equations.

3. BASIC DEFINITIONS:

✔ An **algebraic expression** is a combination of letters, numbers and, at least one arithmetic operation. These expressions are used to translate sentences form the ordinary language to the algebraic language.

✔ The **numeric value** of an algebraic expression is the number that is obtained when we replace the variables with numbers and then, we do all operations.

Example 1: Write the algebraic expression and evaluate it if x=4.
The double of a number: 2x. If x=4, then its value is 2·(4) = 8.

✔ An **algebraic equality** consist of two algebraic expression and an equal sign (=) between them. They can be of two kinds:

- **Identity**: It is true for any value of their letters.
- **Equation**: It isn't true for any value of their letters.

Example 2: Classify the following algebraic equalities in identities and equations.

a) $(x-2)^2 = x^2 - 4$

If $x = 2 \rightarrow (2-2)^2 = 2^2 - 4 \rightarrow 0 = 0$

If $x = 0 \rightarrow (0-2)^2 = 0^2 - 4 \rightarrow 4 \neq -4$

This equality is only true for x = 2, then it is an equation.

b) $(x-2)^2 = x^2 - 4x + 4$

If $x = 2 \rightarrow (2-2)^2 = 2^2 - 4\cdot(2) + 4 \rightarrow 0 = 4 - 8 + 4 \rightarrow 0 = 0$

If $x = 0 \rightarrow (0-2)^2 = 0^2 - 4\cdot(0) + 4 \rightarrow 4 = 4$

If we evaluate it with any value, the equality is always true. Then, it is an identity.

NOTE: Do you remember "the notable equalities"? If you don't remember then it's a good moment because they are a very useful example for algebraic identities.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a-b)(a+b) = a^2 - b^2$$

✔ The main elements of an equation are:

- **Member**: It is each one of the two expressions algebraic separated by the equal sign. The expression on the left is called the first member and the expression on the right, the second member.
- **Unknown quantities:** They are the letters whose values are unknown.
- **Degree:** It is the biggest exponent which the unknown quantities have when all the operations have been made.

✔ When you replace a variable with a value and the sentence is true, this value is the **solution** of the equation. **Solving the equation** is finding out these values. Two **equations** are **equivalent** if they have the same solutions.

<u>Example3</u>: Is some of the following numbers solution of the equation $5\cdot(x-2)=2x+2$ *?*

a) x = 2 → 5·(2 – 2) = 2·(2) + 2 → 0 ≠ 6 → x = 2 isn't a solution.

b) *x = 4 → 5·(4 – 2) = 2·(4) + 2 → 10 = 10 → x = 4 is a solution.*

HOMEWORK:

- Textbook: activities 1, 2, 3, 4, 5 on page 76-77; 45 on page 88.
- Revision Worksheet: activities in section 7.1.

4. SIMPLE EQUATIONS:

4.1. ELEMENTAL SIMPLE EQUATIONS:

✔ A **simple equation** is an equation whose degree is one and which can be written as $ax=b$, with a and b real numbers and $a\neq 0$.

This equation has got an only solution: $x=\frac{b}{a}$.

$$ax=b \Leftrightarrow x=\frac{b}{a}$$

Watch it's only possible when $x\neq 0$. If $x=0$ then the equation hasn't got any solution.

✔In general, to solve any sort of simple equation we write it in its general expression ($ax = b$) by means of equivalents equations (to keep the same solution) according to the following rules:

- **Addition (or Subtraction) Principle**: If you add or subtract the same number to each side of an equation, you get an equivalent equation.
- **Multiplication Principle**: If you multiply or divide each side of an equation by the same number (different of zero), you get an equivalent equation.

Applying these rules we obtain an equivalent equation which is very easy to solve. Let's go to see this application in some examples:

<u>*Example 4:*</u> *The equation* $x+4=7$ *is a model of the situation shown above. According the subtraction principle, you can resolve it subtract 4 from each side (it is the same than add – 4):*

$$x+4=7 \Leftrightarrow x+4-4=7-4 \Leftrightarrow x+0=3 \Leftrightarrow x=3.$$

<u>*Example 5:*</u> *The equation* $5x=-30$ *is a model of the situation shown above. According the multiplication principle, you can resolve it divide each side by 5 (it is the same than multiply by 1/5):*

$$5x=-30 \Leftrightarrow \frac{5x}{5}=\frac{-30}{5} \Leftrightarrow x=-6$$

<u>*Example 6:*</u> *To solve the equation* $2x+1=9$ *we need to apply the two principles. Firstly we subtract 1 from each side and then we divide each side by 2.*

$$2x+1=9 \Leftrightarrow 2x+1-1=9-1 \Leftrightarrow 2x=8 \Leftrightarrow \frac{2x}{2}=\frac{8}{2} \Leftrightarrow x=4.$$

✔You can see that these methods are very long and slow to solve some easy simple equation, but they can be reduced to two general rules, based in the previous principles, which are known as **"transposition of terms":**

- If a term is adding (subtracting) in a member of an equation, it will appear subtracting (adding) in the other member.
- If a term is multiplying (dividing) in a side of an equation, it will appear dividing (multiplying) in the other side.

With this news rules the previous examples are reduced to the following:

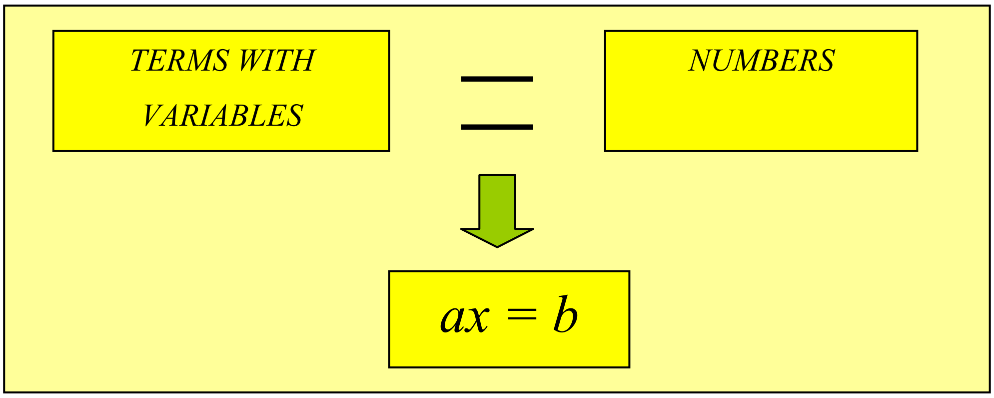
<u>Examples:</u>

a) $x+4=7 \Leftrightarrow x=7-4 \Leftrightarrow x=3$.

b) $5x=-30 \Leftrightarrow x=\frac{-30}{5} \Leftrightarrow x=-6$.

c) $2x+1=9 \Leftrightarrow 2x=9-1 \Leftrightarrow 2x=8 \Leftrightarrow x=\frac{8}{2} \Leftrightarrow x=4$

✔ The principle of addition (subtraction) is also used to deal with **terms which appear with variables**. Normally the terms with variables will be put in the left member and all the number in the other. When they are in the correct member of the equation we operated them until obtaining an only term in every side, in other words until obtaining a simple equation in its standard form.



<u>Example 7:</u> Solve the equation: $x-2x+3x-6=10-2x$

Let's go to change the terms -6 and -2x which appear in the incorrect member using transposition of terms. Then we will operate each member to obtain an elemental equation that we already know to solve.

$$x-2x+3x+2x=10+6 \Leftrightarrow 4x=16 \Leftrightarrow x=\frac{16}{4} \Leftrightarrow x=4$$

HOMEWORK:

- Textbook: activities 10, 12 on page 79; 49, 50 on page 88.
- Revision Worksheet: activities 4, 5 in section 7.2.

4.2. SIMPLE EQUATIONS WITH BRACKETS:

When an equation contains brackets, we need to eliminate them to make the equation easier to solve. The ***Distributive Law*** is used to do this, which says:

$$a \cdot (b+c) = a \cdot b + a \cdot c.$$

After applying this property you will obtain an elemental equation which will be solved like we have just seen in the previous paragraph.

Example 8: *Solve the equation* $x - 5(x+2) = 6x$.

Firstly, to eliminate the brackets we apply the distributive law (be careful with the signs in brackets) and obtain:

$$x - 5x - 5 \cdot 2 = 6x \Leftrightarrow x - 5x - 10 = 6x.$$

Now we need to get an elemental equation, in other words all the terms with x have to be in a member and the number in the other. We use "transposition of terms" to get it:

$$x - 5x - 6x = 10$$

Then we operate in every member to get an elemental equation and solve it:

$$-9x = 10 \Leftrightarrow x = \frac{10}{-9} \Leftrightarrow x = -\frac{10}{9}$$

HOMEWORK:

- Textbook: activity 52 on page 89.
- Revision Worksheet: activities 6, 7 in section 7.2.

4.3. SIMPLE EQUATIONS WITH FRACTIONS:

When an equation contains fractions, it makes it easier to solve them when the fractions aren't there. The ***Multiplication Principle*** is used to do this, using like factor the **lowest common multiple (LCM)** of all the denominators that appear in the equation. After doing this you will obtain an equation with brackets which will be solved like the previous paragraph explains.

Example 9: *Solve the equation* $\frac{x-1}{2} - \frac{5}{4} = \frac{5}{2}x$.

Firstly, to eliminate the denominators we multiply both sides of the equation by the LCM of the denominators, 4 in this case.

$$4 \cdot \left(\frac{x-1}{2} - \frac{5}{4} \right) = 4 \cdot \frac{5}{2} x$$

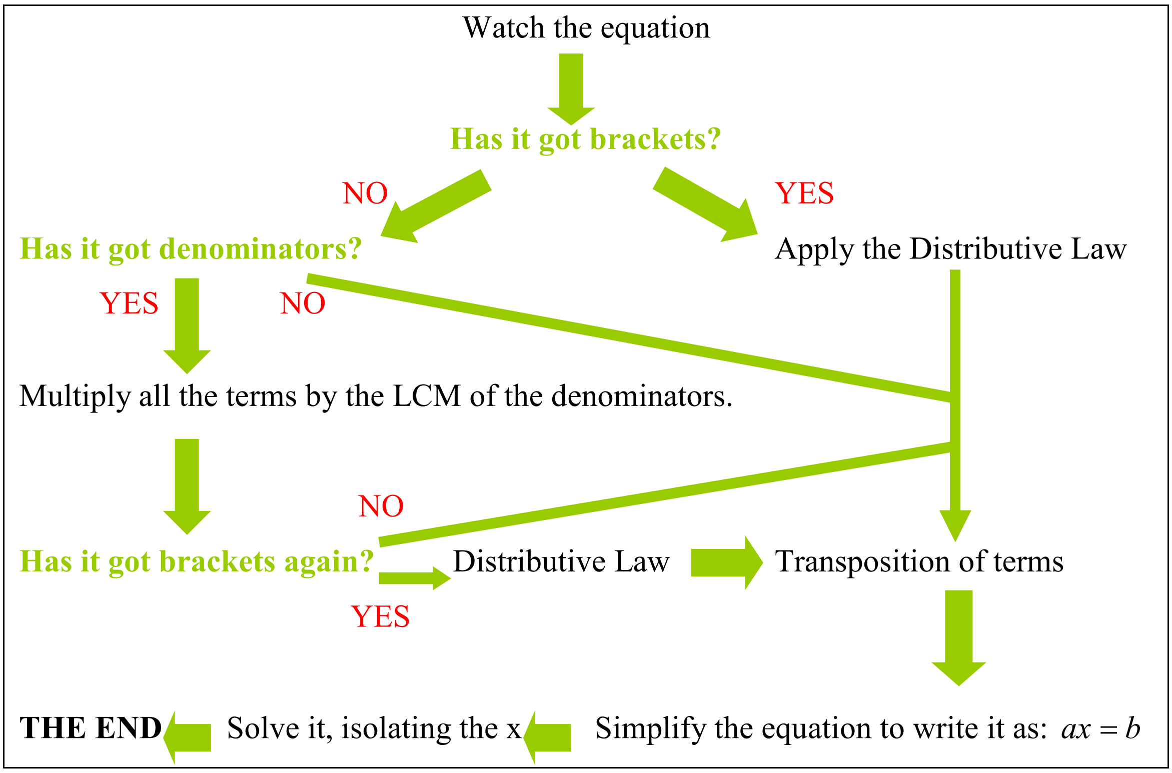
Now you use the Distributive Law and simplify to get an equation with no fractions.

$$\frac{4(x-1)}{2} - \frac{20}{4} = \frac{20}{2} x \Leftrightarrow 2(x-1) - 5 = 10x$$

After following the same process than in the previous example, you will obtain the solution:

$$2x - 2 - 5 = 10x \Leftrightarrow 2x - 10x = 2 + 5 \Leftrightarrow -8x = 7 \Leftrightarrow x = \frac{7}{-8} \Leftrightarrow x = -\frac{7}{8}$$

✔After learning all the methods **to solve every kind of simple equations** we can do an outline to sum up all this contents:



HOMEWORK:

- Textbook: activities 16, 17 on page 80; 56, 57 on page 88.
- Revision Worksheet: activities 8, 9 in section 7.2.

5. QUADRATIC EQUATIONS:

✔ A **quadratic equation** with an unknown quantity is an algebraic equality which can be written in the following form, in which *a, b* and *c* real numbers and $a \neq 0$.

$$ax^2 + bx + c = 0$$

Depending of the values of their coefficients, they can be of two kinds:

- **Complete**: If $b \neq 0$ y $c \neq 0$.
- **Incomplete**: If $b = 0$ o $c = 0$.

Let's go to see the way of solving each one of them.

5.1. COMPLETE QUADRATIC EQUATIONS:

The solutions of a complete quadratic equation are obtained using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The sign $\pm$ indicates that there are two solutions.

✔The number $b^2 - 4ac$ is represented by the symbol Δ and **the number of solutions** of the equation depends on its sign:

- If $\Delta > 0$, then the equation has got **two solutions**:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad y \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- If $\Delta = 0$, then the equation has only got **one solution**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$$

- If $\Delta < 0$, then the equation hasn't got **any solution**, because the quadratic root $\sqrt{b^2 - 4ac}$ can't be calculated.

<u>Example 10</u>: Determine the number of solutions of the following quadratic equations and then calculate their values.

a) $x^2 - 7x + 12 = 0$

$$a=1, b=-7, c=12 \Rightarrow b^2-4ac=(-7)^2-4\cdot 1\cdot 12=49-48=1>0 \Rightarrow 2 solutions$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-7)\pm\sqrt{1}}{2\cdot 1}=\frac{7\pm 1}{2}=\begin{cases} x_1=\frac{7+1}{2}=\frac{8}{2}=4 \\ x_2=\frac{7-1}{2}=\frac{6}{2}=3 \end{cases}$$

b) $3x^2-x+1=0$

$$a=3, b=-1, c=1 \Rightarrow b^2-4ac=(-1)^2-4\cdot 3\cdot 1=1-12=-11<0 \Rightarrow 0 solutions$$

c) $4x^2+4x+1=0$

$$a=4, b=4, c=1 \Rightarrow b^2-4ac=4^2-4\cdot 4\cdot 1=16-16=0 \Rightarrow 1 solution$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-4\pm\sqrt{0}}{2\cdot 4}=\frac{-4}{8}=-\frac{1}{2}$$

✔Bear in mind that this formula can be applied when the equation appear in its standard form $ax^2+bx+c=0$, so if you have a **quadratic equation with other appearance** you need to change it using the methods that we have studied for the simple equations (distributive law, multiplication by the LCM, transposition of terms…).

Example 11: Solve the equation: $(x-2)^2-3x=2x^2$

Let's go to develop the notable identity and then apply the methods of transposition of terms to get its general expression:

$$x^2-4x+4-3x=2x^2 \Leftrightarrow x^2-4x+4-3x-2x^2=0 \Leftrightarrow -x^2-7x+4=0$$

Now you can apply the general formula to solve this complete quadratic equation in which $a=-1, b=-7, c=4$.

HOMEWORK:

- Textbook: activities 19, 20, 22 on page 81; 61, 62 on page 90.
- Revision worksheet: activity 10 in section 7.3.

5.2. INCOMPLETE QUADRATIC EQUATIONS:

✔**CASE 1:** If $b = 0$, in other words an equation in the form: $ax^2 + c = 0$

To solve it we have to get that x^2 is isolated in the first member. We will get it using the two principles of transposition of terms. After applying them, we will try to calculate the square root of the second member and we will find two possible situations:

$$ax^2 + c = 0 \Leftrightarrow ax^2 = -c \Leftrightarrow x^2 = -\frac{c}{a} \Rightarrow \begin{cases} If\ -\frac{c}{a} > 0 \Rightarrow 2solutions : x = \pm\sqrt{-\frac{c}{a}} \\ If\ -\frac{c}{a} < 0 \Rightarrow 0solutions \end{cases}$$

<u>Example 12</u>: Solve the equation: $3x^2 - 27 = 0$

$$3x^2 - 27 = 0 \Leftrightarrow 3x^2 = 27 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3$$

✔**CASE 2:** If $c = 0$, in other words an equation in the form: $ax^2 + bx = 0$

To solve it we find the common factor of the first member to write it as the product of two algebraic expressions. Now we apply *The Principle of Zero Products* which says: "If a and b are real numbers, and a·b = 0, either a, b, or both equal 0". Then we will find the two solutions of our equation.

$$ax^2 + cx = 0 \Leftrightarrow x \cdot (ax + b) = 0 \Leftrightarrow \begin{cases} x = 0 \\ or \\ ax + b = 0 \Leftrightarrow ax = -b \Leftrightarrow x = -\frac{b}{a} \end{cases}$$

<u>Example 13</u>: Solve the equation: $3x^2 - 2x = 0$

$$3x^2 - 2x = 0 \Leftrightarrow x \cdot (3x - 2) = 0 \Leftrightarrow \begin{cases} x = 0 \\ 3x - 2 = 0 \Leftrightarrow x = \frac{3}{2} \end{cases}$$

Sometimes we can find that the common factor isn't only the variable x. In this case we should take the complete common factor, a number and the x, to simplify the calculations:

<u>Example 14</u>: Solve the equation: $12x^2 - 36x = 0$

$$12x^2 - 36x = 0 \Leftrightarrow 12x \cdot (x - 3) = 0 \Leftrightarrow \begin{cases} 12x = 0 \Leftrightarrow x = 0 \\ x - 3 = 0 \Leftrightarrow x = 3 \end{cases}$$

✔**CASE 3:** If $b = 0 \;\; y \;\; c = 0$, in other words an equation in the form: $\boxed{ax^2 = 0}$

This is the easiest situation, because it's clear that the only solution of this type of equation is $\boxed{x = 0}$.

HOMEWORK:

- Textbook: Activities 26, 27 on page 83; 65 on page 91.
- Revision worksheet: activities 11, 12, 13 in section 7.3.
- Interactive activity "Algebra Four" in **http://www.shodor.org/interactivate** (Look for "learners", click on "Activities" and choose "Algebra Four").

6. EQUATIONS IN REAL-LIFE PROBLEMS:

The word **"problem"** usually strike fear into the hearts of math students but they aren't really so bad. To solve these problems more easily you can follow some steps which will help you a lot:

1. Identify the unknown quantity of the problem and using a variable to represent it.

↓

2. Look for words in the problems that can be described using numbers and mathematic symbols.

↓

3. Use algebra to write an equation that means the same thing as the original problem.

↓

4. Solve the equation.

↓

5. Check that the solution makes sense for the real value which it describes.

✔**Problems with only one unknown quantity:** In this kind of problem is very easy determining what is the variable x and the difficulty is in translating the situation from the ordinary language to the algebraic speech. Let's go to see some examples:

<u>Example 15</u>: *Find a number so that the sum of twice this number and 13 is 75.*

Firstly we use the letter x to describe the number which we are looking for.

Now we have to translate the wording of the problem using our knowledge about algebraic language: The word "is" means "equal", the word "and" means "plus" and the rest of the sentence can be written very easily because its expression is completely mathematic. Then our problem entails solving the simple equation: $2x+13=75$.

Using the two principles of transposition of terms we can obtain the number which is asked by the problem:

$$2x+13=75 \Leftrightarrow 2x=75-13 \Leftrightarrow 2x=62 \Leftrightarrow x=\frac{62}{2} \Leftrightarrow x=31$$

Watch that the number 31 is a correct solution for our problem because it asks us every number without any kind of condition.

<u>Example 16</u>: *Find a number which decreased by 18 is 5 times its opposite.*

Let's go to call x to our number. Again, you look for mathematic symbols that describe the words that appear in the wording: "Is" means "equal", and "decreased by" means "minus". Also "opposite" always means "negative". Keeping that information in mind, the equation that describes the problem can be written just like the following: $x-18=5\cdot(-x)$

Now we multiple out and solving the simple equation that we have written:

$$x-18=5\cdot(-x) \Leftrightarrow x-18=-5x \Leftrightarrow x+5x=18 \Leftrightarrow 6x=18 \Leftrightarrow x=\frac{18}{6} \Leftrightarrow x=3$$

And clearly the number 3 is a correct solution for our example.

<u>Example 17</u>: *If the area of a square is 36 square centimetres, determine the value of its side.*

We use the variable x to describe the unknown quantity, in other words x is the value of the square's side.

Now we remember that the area of a square is the square of its side, therefore the problem asks to find a number whose square is 36. In algebraic language it can be written like the following: $x^2 = 36$

Watch that it's an incomplete quadratic equation because there isn't term with grade one. We have already learnt to solve it:

$$x^2 = 36 \Leftrightarrow x = \pm\sqrt{36} \Leftrightarrow x = \pm 6$$

This equation has got two solutions, but remembers that x is a length, and therefore it can't be a negative number. Then only one of these two values makes sense for our problem, exactly the square's side is 6 centimetres long.

✔**Problems with several unknown quantities:** In this kind of problem is more complicated to name all the quantities that appear, because we only can use a letter. So the most important thing in these problems is finding the relations among the quantities to write by means of algebraic expressions with x. Let's go to see some examples:

Example 18: The sum of three times a whole number and 7 units is the same value than the sum of the double of its consecutive number and 3 units. Determine this number.

Firstly we have to determine the names for our variables. Let's go to name x to the main number and then its consecutive will be named x+1.

Now let's go to write in algebraic speech the two sums that appear in the wording:

- *The sum of three times a number and 7 units: 3x+7*
- *The sum of the double of its consecutive number and 3 units: 2(x+1)+3*

The problem says that these two quantities are the same, so we equal them to obtain our equation: $3x+7=2(x+1)+3$

It's a simple equation with brackets, which we know to solve:

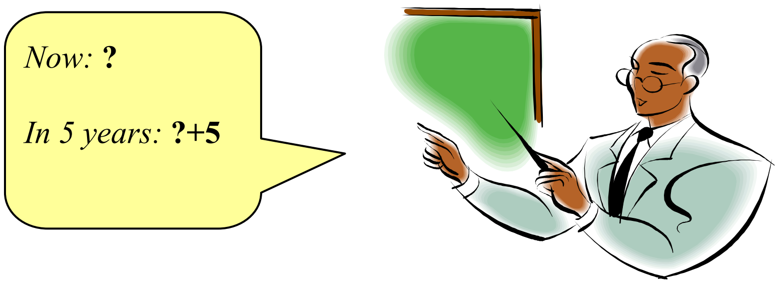
$$3x+7=2(x+1)+3 \Leftrightarrow 3x+7=2x+2+3 \Leftrightarrow 3x-2x=2+3-7 \Leftrightarrow x=-2$$

And –2 is a whole number, so that it's a correct solution for this problem.

<u>Example 19</u>: My sister's age is three times the mine and in five years´ time her age will be the double that the mine. How old are we?

We name "x" to my age and then my sister's age has to be "3x".

Now we think about how write our ages in five years´ time:



In our case: I will be x+5 and my sister 3x+5. And my sister's age in that moment will be the double that the mine, soothe value x will satisfy the following equation with brackets: $3x+5=2(x+5)$.

$$3x+5=2(x+5) \Leftrightarrow 3x+5=2x+10 \Leftrightarrow 3x-2x=10-5 \Leftrightarrow x=5$$

Therefore, according to the names that we took for our ages, I'm 5 years old and my sister is 15 years old (because 3·5=15).

HOMEWORK:

- Textbook: Activities 29, 30, 31, 33, 34, 35, 36, 37 on page 85; 72, 73, 74, 76 on page 91; 79, 80 on page 92; 89, 90, 91 on page 93.
- Revision worksheet: activities in section 7.4 and 7.5.

7. REVISION WORKSHEET:

7.1. DEALING WITH THE BASIC ALGEBRAIC ELEMENTS:

ACTIVITY 1: Indicate if the following equalities are identities or equations:

a) $(x+5)^2 = x^2 + 10x + 25$

b) $3(x-2) = 2x - 6$

c) $(x+3)(x-4) = x^2 - x - 12$

d) $x^3 - 1 = (x-1)(x^2 + x + 1)$

e) $(2x^2 - 3)^2 = 2x^4 - 12x^2 + 9$

f) $x(2-x) + x = -x^2 + 3x$

> Have to operate every member separately and then check if the same algebraic expression is obtained in both sides.

ACTIVITY 2: Determine if some of these numbers: 2, – 3, 0, $\frac{1}{2}$ y 5, is solution of some of the following equations, calculating the numeric value of each one of their members:

a) $4x - 3 = 2x + 7$

b) $6(x-1) = x + 3(x-2)$

c) $2x + 1 = x + \frac{3}{2}$

d) $x^2 - 3x = 10 + x$

ACTIVITY 3: Determine three equivalent equations for each one of the following equations:

a) $5x = -2$

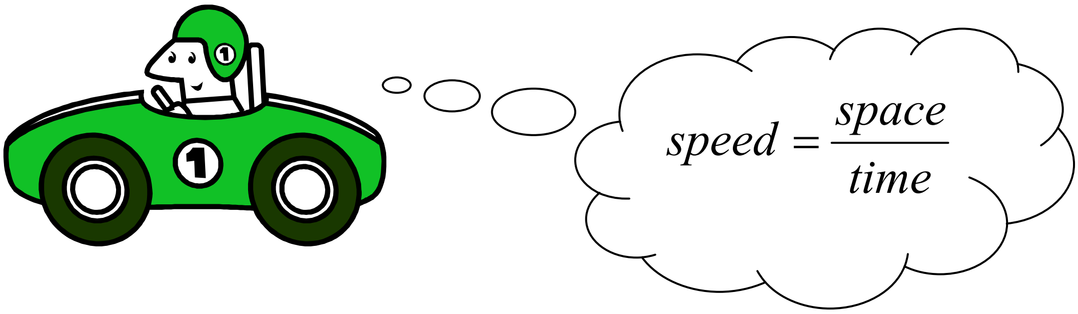
b) $2x - 1 = 3x + 2$

Indications:

- For calculating the first equivalent equation apply only the addition principle, for the second one only the multiplication principle and for the third one the two principles together.
- Write the members of each one of the six equations that you obtain in the most simplified form that it's possible.

7.2. DEALING WITH SIMPLE EQUATIONS:

ACTIVITY 4: Watch the formula that links speed with time and space and use the transposition of terms to find the value of the element which lacks in each case:



a) The average speed of a car that go round 140 Km in three hours.
b) The amount of minutes that lasted a cyclist for going round a length of 30 Km if his average speed was 15 km an hour.
c) The distance between two cities if I take 30 minutes in going round it with an average speed of 8 Km/h.

ACTIVITY 5: Determine the value of "a" in each one of the following equations so that the indicated conditions are complied by this value:

a) The solution of the equation $ax = 18$ is $x = -6$.
b) The solution of the equation $2x = a$ is $x = 8$.
c) The solution of the equation $x - 2 = a$ is $x = -3$
d) The solution of the equation $x - a = 5$ is $x = 4$.

ACTIVITY 6: Write equations that comply with the following conditions:

1. Its degree is one and its solution is the number 3.
2. It's equivalent to the equation $2(x-1) - 3(2x+3) = 3 - (2x-5)$.
3. It has got brackets and is equivalent to the equation $3x = 5$.

ACTIVITY 7: Solve the following simple equations with brackets:

a) $5(x-3) = 10$
b) $1 - 3x = 4x + 5 - (4 - x)$
c) $15x - 5(x-1) = 120 - 5x$
d) $7 + 3(2+x) - 3x = 9 + 2x$

ACTIVITY 8: Solve the following simple equations with fractions:

a) $\frac{2x-31}{6}=\frac{x-3}{4}$

b) $-2x+5=\frac{x+3}{2}$

c) $2(x-1)=\frac{x-3}{2}-\frac{1-2x}{6}$

d) $\frac{x-2}{2}-\frac{x-3}{3}-\frac{x-4}{4}=0$

e) $\frac{x-3}{7}+\frac{2x+1}{2}=\frac{3}{14}$

f) $\frac{1}{8}(x-2)-\frac{3}{4}(x+6)+x=-1$

ACTIVITY 9: Solve the following equations:

a) $(x+3)(x-2)=x(x-36)$

b) $(2x+1)^2-(2x-1)^2=100$

c) $\frac{(x-1)(2x+3)}{5}-(x^2-3)=\frac{x(x+4)}{2}-\frac{11x^2}{10}$

d) $(x-2)(x+2)=1-\frac{x(2-3x)}{3}$

When you operate these equations you'll watch they are simple equations although they don't seem to be it because of their terms with degree two.
Remember:
Appearances can be deceptive...

7.3. DEALING WITH QUADRATIC EQUATIONS:

ACTIVITY 10: Indicate the value of the coefficients and the number of solutions of each one of the following equations without solving them:

a) $-3x^2+7x-1=5$. *b)* $x^2-3x=2x+3$. *c)* $x^2-6x+9=0$.

ACTIVITY 11: Write equations that comply with the following conditions:

1. Its degree is two and its only solution is the number 0.
2. It's an incomplete quadratic equation whose solutions are + 6 y – 6.
3. It's an incomplete quadratic equation whose solutions are 0 y 2.
4. It's an incomplete quadratic equation that hasn't got any solution.

ACTIVITY 12: Find some value of "a" so that these quadratic equations comply with the conditions which are indicated in each case:

a) The equation $ax^2-4x+2=0$ has got two solutions.

b) The equation $-x^2+ax-1=0$ has got only one solution.

c) The equation $x^2-a=0$ hasn't got any solution.

d) The solutions of $x^2+ax=0$ are 0 and 2.

<u>ACTIVITY 13:</u> Solve the following quadratic equations:

a) $(x-2)(x+2)+10=2x(x-1)+(3x+1)^2$

b) $(2x^2-3)^2-6=-1-4x^2(2-x^2)$

c) $\dfrac{(1-x)(3-2x)}{6}=-\dfrac{x(2-3x)}{3}$

d) $\dfrac{2x^4}{5}-\dfrac{3-x^2}{2}=1-\dfrac{3-4x^4}{10}$

e) $x^3+2x^2=x(x^2-3)$

Be careful with these equations because they are very far from the standard form of a quadratic equation and a lot of changes are necessary to transform them.

7.4. SOLVING REAL-LIFE PROBLEMS:

<u>ACTIVITY 14:</u> A girl is 26 years old and her mother is 49. How many years the mother's age was the double than the girl's age ago?

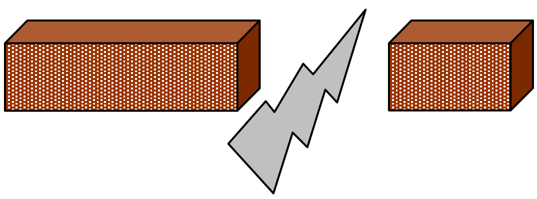
<u>ACTIVITY 15:</u> A father's age is three times her son's one and in 16 years´ time it will be only the double. How old are they?

<u>ACTIVITY 16:</u> A mother's age is three times her daughter's one. If the mother was 30 years younger and the daughter 8 years older, then both would be the same age. Determine how old they are.

<u>ACTIVITY 17:</u> The sum of a number and 9 divided by 5 is the same value as the subtraction of the number and 9 divided by 2. What is this number?

<u>ACTIVITY 18:</u> A man cut two pieces from a piece of wood so that one is a half of the total and the other is only a quarter. If the two pieces together are 360 cm long, how long is the complete piece?

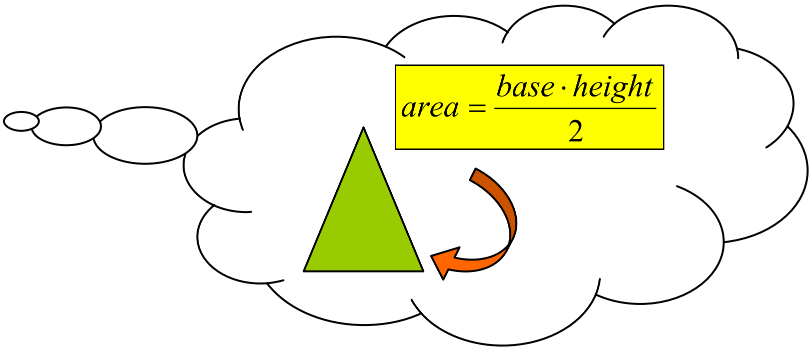
A HALF + A QUARTER = 360 CM



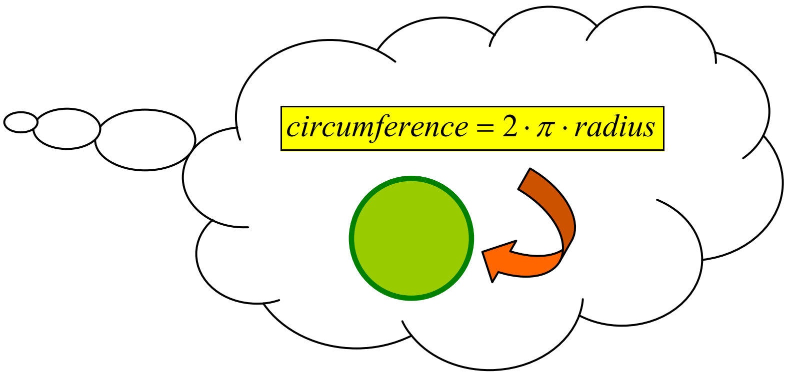
ACTIVITY 19: A boy has got 18 coins, some ones are of 1 € and other ones are of 2 €. Determine the number of coins of each sort if all together sum 26 €.

ACTIVITY 20: An exam consists of 20 questions. Every correct answer sums 3 points and every question which is wrong answered or unanswered subtracts 2 points. If a pupil gets 30 points in his exam, how many questions has he answered correctly?

ACTIVITY 21: If the area of a triangle is 400 cm^2 and its height is three times its base, how long are its dimensions?



ACTIVITY 22: The radius of a circle is 8 cm long. How many centimetres more should the radius be long to obtain a new circle whose circumference is three times the circumference of the first circle?



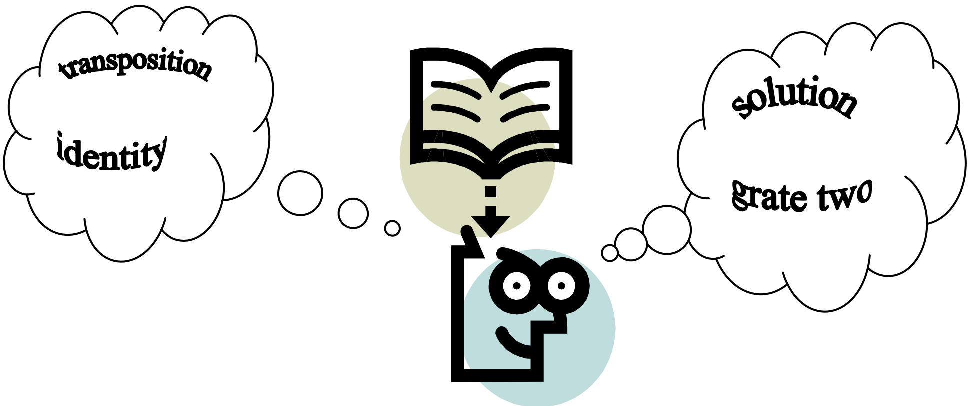
ACTIVITY 23: Is it possible that the multiplication of two consecutive numbers is equal to 24? Explain your answer by means of an equation.

¿¿¿¿ (A NUMBER) · (ITS CONSECUTIVE) = 24 ????

7.5. REVISING VOCABULARY AND THEORETIC KNOWLEDGE:

ACTIVITY 24: Fill the blanks with the correct word among the ones that appear in the speech bubble below:

1. All the quadratic equations have got ____________.
2. If we calculate the numeric value of the two members of an equation for a number and the same value is obtained in each side, then this number is a __________ of the equation.
3. If operating the two members of an algebraic equality, the same algebraic expression is obtained then this is an _________.
4. The method used to get the standard expression of an equation is called ______.



ACTIVITY 25: Indicate if the following sentences are true or false, giving a reason for your answers:

1. If a simple equation is satisfied for two different numbers then is an identity.
2. A quadratic equation has always got some solution.
3. All the values that you obtain when you solve the equation associated to a problem are solutions for the problem.
4. If both members of an equation with fractions are multiplied by the LCM of the denominators then the new equation has got the same solutions that the initial.

<u>ACTIVITY 26:</u> Fill the following horizontal lines with the correct words whose definitions are given below:

E								
Q								
U								
A								
T								
I								
O								
N								
S								

1. It's the sign that appears between the two members of an equation.
2. This name is given to the equations whose degree is two.
3. We use the letter x to design this kind of quantities.
4. It's the branch of mathematics concerning the study of structure, relation, and quantity.
5. It's the number of solutions of the equation: $x^2 + x = 0$.
6. It's an equation that any number is solution.
7. It's the degree of a simple equation.
8. The value of an algebraic expression when its letters are replaced with numbers.
9. It's the name of the values which satisfy an equation when its numeric value is calculated.

<u>ACTIVITY 27:</u> All the following cards will be cut out and handed out among all your classmates by your teacher. When you receive the your, solve the equation and find other person who has got a card with an equivalent equation.

$x^2 - 3x = 0$	$(x-3)^2 = 9 - 3x$	$3x^2 - 20 = 7$	$(x-1)^2 + 2x = 10$
$\frac{x-3}{2} = \frac{x+1}{3}$	$5x - 10 = 4x + 1$	$x - 3 = 2(x+1) - x$	$\frac{x-2}{5} = \frac{2x-3}{10}$
$3(x+2) = 2(x+5)$	$\frac{2x}{3} + 1 = \frac{1}{3}$	$x - 3 = 2x - 7$	$7x - 2 = -9$
$\frac{x}{5} + 1 = \frac{2x-3}{5}$	$x - 3 = 10$	$\frac{x-3}{2} = 5$	$x - 4 = 12 - x$
$x - 3 = 2(x+1)$	$2x = -8$	$2(3x+1) = 4x - 6$	$x + 2 = -3$